

Sensitive predators and enduring preys in an ecosystem driven by correlated noises

Wei-Rong Zhong, Yuan-Zhi Shao,* and Zhen-Hui He

Department of Physics, Sun Yat-sen University, 510275 Guangzhou, People's Republic of China

We investigate the Volterra ecosystem driven by correlated noises. The competition of the predators induces an increasing in population density of the predators. The competition of the preys, however, leads the predators to decay. The predators may have better stability under strong correlated noises. The predators undergo a sensitivity to a random environment, whereas the preys exhibit a surprising endurance to this stochasticity.

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Predator-prey ecosystems are one of the most amazing models in interacting population biology [1-3]. The prey-predator model and its derived ones are applied in a wide range of fields, for example, tumors, measles, epidemics and food chains [4-6]. Though predator-prey model suggests that the simple interactions can result in periodic behavior of the population, it is still an unrealistic assumption. To fill this deficiency, scientists struggle to find more realistic approaches. Stochastic models are so far better selections, including individual level model (ILM)[7] and two-species stochastic population model (SPM) [8-10]. The anterior probes a situation when the number of individuals is finite, and the later describes a strictly infinite one.

Surprising phenomena emerge sometimes just because there exists correlation or collaboration; one of the most well known example is the self-organized behavior [11]. The population in ecosystems, including prey and predators, are affected by simple stochastic processes of mortality, reproduction, and predation. Differential equations with stochastic components match their realities better than the deterministic ones. Due to the difficulty and complexity in mathematical analysis, previous researches in prey-predator models only consider independent noises [8-10]. Regrettably, the correlated noises, which are more realistic tentation to depict true nature than independent ones, are easily left in the basket. We all know that stochastic processes in ecosystems may come from the same origin like disaster, thus it is reasonable to consider the correlation between noises. In this paper, we focus on the two-species model with correlated stochastic components for the first time. The correlated noises we consider here, which distinguish from independent ones Cai and Lin have concerned with in their latest paper [8], induce some novel phenomena in a prey-predator ecosystem that are not found before.

We study two-species models to elucidate the mechanism of interesting influence of correlated noises on the stochastic dynamical ecosystems. Consider Volterra's model with prey resource limitation, described by the

stochastic differential equation

$$\frac{dX}{dt} = r_1 X \left(1 - \frac{X}{K}\right) - k_1 XY + X \left(1 - \frac{X}{K}\right) \xi(t), \quad (1)$$

$$\frac{dY}{dt} = -r_2 Y + k_2 XY + Y \eta(t), \quad (2)$$

where $X \geq 0$ and $Y \geq 0$ denote the densities of individuals in a population density of preys and predators, respectively. In the absence of noise, they will evolve into a stable value (X_0, Y_0) as $t \rightarrow \infty$. r_1, r_2 are the corresponding reproduction rate. K is the capacity of prey population, that is, the population size for $t \rightarrow \infty$ if $Y \equiv 0$. The coefficient k_1 quantifies the impact which an individual predator has on the reproduction rate of an individual prey. Conversely, k_2 gives the impact which an individual prey has on the reproduction rate of a predator. $\xi(t), \eta(t)$ are correlated, Gaussian white noises satisfying

$$\langle \xi(t) \xi(t') \rangle = 2M_1 \delta(t - t'), \quad (3)$$

$$\langle \eta(t) \eta(t') \rangle = 2M_2 \delta(t - t'), \quad (4)$$

$$\langle \xi(t) \eta(t') \rangle = 2\lambda \sqrt{M_1 M_2} \delta(t - t'), \quad (5)$$

in which M_1, M_2 are the intensities of noises; λ , ranges from zero to one, denotes the correlation coefficient between $\xi(t)$ and $\eta(t)$, $\delta(t - t')$ is Dirac delta function under different moments.

An approach to describing stochastic noise is using three important ideas: samples, events, and probability [12]. For example, consider a large population, say N , of preys. These are the samples. The events we consider are the collection of all possible aggregations of these preys. The probability of an event is simple $(1/N)$ times the number of samples in the event. Therefore, our notation for the stationary probability distribution (SPD) of an event X (a population density of the preys) or Y (a population density of the predators) is

$$P(X) = \frac{\text{Number of preys in } X}{N_1}, \quad (6)$$

$$P(Y) = \frac{\text{Number of predators in } Y}{N_2}, \quad (7)$$

in which N_1, N_2 are all preys and predators, respectively. Consider $P(X)$ and $P(Y)$ are independent, the joint stationary probability distribution

*Corresponding Author; Electronic address: stssyz@zsu.edu.cn

FIG. 1: Joint stationary probability distributions under different noise intensities. The parameters are $M_1 = 0.05$, $\lambda = 0.0$, (a) – (d) $M_2 = 0.1, 0.5, 2.0, 5.0$.

(JSPD) of (X, Y) is $P(X, Y) = P(X)P(Y)$ satisfying $\int_0^\infty \int_0^\infty P(X, Y) dX dY = 1$.

In general, the probability distribution is still a qualitative parameter, though noise-induced transition can be depicted by observing its changing trend. Here we define a quantitative parameter, i.e. the mean density of population, which is useful to display the influence of noises on the prey-predator ecosystems. The mean densities of the preys and predators are respectively written as

$$\langle X \rangle = \int_0^\infty \int_0^\infty X P(X, Y) dX dY, \quad (8)$$

$$\langle Y \rangle = \int_0^\infty \int_0^\infty Y P(X, Y) dX dY, \quad (9)$$

and their variances are $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$, and $\sigma_Y^2 = \langle Y^2 \rangle - \langle Y \rangle^2$, respectively.

One of the approaches to solve Eq.(1) and (2) is deriving the equivalent stationary Fokker-Planck equation [8], but that is possible only toward two non-correlated noises. Once consider the correlation of noises, it is difficult to derive and solve the Fokker-Planck equation. Here we apply a real-time simulation of Eq.(1) and (2) to obtain the joint stationary probability distributions of the preys and predators densities. JSPD is useful to represent the influence of noises on the preys and predators. The quantitative parameters are also calculated for deep analysis. For example, the mean density of the predators is used to display their long-term behavior.

Figure 1 shows the peak of the joint stationary probability distributions drops and shifts to position $(0, 0)$ with M_2 increasing. With an increase in M_1 , similar effects are observed, and they are not plotted here for simplicity. Obviously, lower peak height and more large preys and predators densities mean less system stability.

FIG. 2: Relationship of mean densities of the preys and predators with the noises intensities as $\lambda = 0.0$. (a) $M_1 = 0.05$, (b) $M_2 = 0.1$.

It is comprehensible that noises can induce un-stability. Consider a long term, whether the noises will do good to the preys and predators or not is still not known. So quantitative parameters, e.g. the mean densities of the preys and predators are helpful to do that. Only according to the JSPD, it looks as if there are no differences between the influence of noises on the preys and those on the predators. Figure 2, however, illustrates their distinctions. The average predator density increases with M_2 , and decreases with M_1 . Obviously, $\eta(t)$ does good to the predators, but $\xi(t)$ does not. Provided that the former is regarded as the predators competition and the latter as the preys competition, it is not difficult to comprehend why predators benefit from their competitions, and why not from the prey competitions. Surprisingly, the preys densities properly maintain their intactness in spite of the changing in M_1 or M_2 . Why do they happen like that? Here we suggested an important reason, which is easily understood from Eq.(1) and (2): both of the growth rate and source limitation keep the prey's stability, whereas the only death rate can not keeps the predator's stability. Perhaps this is why the predators are more sensitive to the stochastic noises than the preys.

The counterpart distinctions induced by the correlation are shown in Fig.3 and Fig.4. Figure 3 displays that the stationary probability distributions of the prey does not change with the noise correlation. The stationary probability distributions have the same fitting curve under different correlations. Their variances are also invariable (see the inset of Fig.3). This indicates that the correlation has little effect on the preys. Unlike what have happened on the preys, figure 4 shows that the peak height of the stationary probability distributions of the predators increases with the correlation (see the corresponding fitting curves), and its variance, shown in

FIG. 3: Stationary probability distributions of the preys under different correlations. The parameters are $M_1 = 0.05$, $M_2 = 0.1$. Inset: Variances of SPD against the correlations, λ .

FIG. 4: Stationary probability distributions of the predator under different correlations. The parameters are $M_1 = 0.05$, $M_2 = 0.1$. Inset: Variances of SPD against the correlations, λ .

the inset of Fig.4, decreases with the correlation. Like what we have mentioned above, $\xi(t)$ and $\eta(t)$ refer to the preys and predators competitions, respectively. We suggest that the correlation between noises refers to the synchronous components of the competitions, and thus it is possible for the predators to maintain more stable SPD provided that they adjust their competition to those of the preys.

In summary, the correlation or collaboration, which exists in numerous dynamical systems, can cause interesting responses of the prey-predator ecosystems to noises. Due to the support of their growth and resource limitation, the preys undergo perfect endurance to the external noises. Conversely, the predators are not only sensitive to the noise intensity, but also impressible to the noise correlation. Strong correlated noises can lead the stationary probability distributions of the predators to stability. Our model is expected to support a fact: the complexities of biosystems originate from their collaborations and correlations.

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